Numerical Model of the Dynamic Response of 3D Conducting Structures with Magnetic Damping

A. Portone[1], G. Rubinacci[2], and P. Testoni[1]

[1] Fusion for Energy, Barcelona Spain, Alfredo.Portone@f4e.europa.eu, Pietro.Testoni@f4e.europa.eu
[2]Department of Electrical Eng. and Information Technologies, Università di Napoli Federico II, Napoli, Italy, rubinacci@unina.it

In this paper, we present a numerical model for the solution of the classic electromagneto-mechanical coupled problem. The dynamical behavior of conducting structures in the presence of a strong magnetic damping was the subject of a high scientific interest in the past, leading to several computational models with experimental validation, mainly related to thin shell structures. We extend this approach to the treatment of three-dimensional conducting structures. To this purpose, we couple a very effective 3D integral formulation in terms of the current density to the 3D dynamical model of the conducting structures. The formulation is validated against the experimental results of the TEAM-16 benchmark problem. In the full paper, the importance of the magnetic damping will be assessed with reference to the analysis of dynamic response of the vacuum vessel of a fusion device under the strong Lorentz forces due to the plasma current disruption. The complex geometry of the vacuum vessel represents a real challenging problem in this frame.

***Index Terms*— Electromagneto-mechanical coupling, Eddy currents, Magnetic Damping, Integral formulation, Edge elements.**

I. Introduction

THE DYNAMICAL behavior of conductive structures in the presence of a strong magnetic damping was the subject of a noticeable scientific interest in the past, leading to several computational models with experimental validation, mainly related to thin shell structures [1], [2]. More recently, a similar coupled problem has been analyzed in the frame of innovative applications [3]. One of the drawbacks of the differential approaches is the possible numerical instability due to the presence of the velocity term in the governing equations written in the Euler coordinate system. This term disappears if a Lagrangian approach [2], [4], [5] is adopted. However, in principle one has to take into account the deformation of the structure during the transient. This leads to a large computational effort, since one has to re-mesh the domain and update the coefficient matrices at every time-step. For this reason, an alternative approach based on an integral formulation of the electromagnetic problem can be a valuable alternative. Suitable sparsification and parallelization algorithms, as already proved in the past, can conveniently solve the problems arising due to the presence of full matrices. The proposed approach, detailed in Section II, is applied to the solution of TEAM-16 benchmark problem [1]. In the full paper, the effectiveness of the method will be highlighted with a complex example of interest for the design of a nuclear fusion device.

II. Numerical Model

We assume a 3D conducting domain V_c in the presence of time varying electromagnetic sources. Pulsed eddy currents are induced giving rise to Lorentz forces in the conducting structure. In the limit of small displacements, after discretization, the subsequent deformation of the specimen can be obtained as the solution of the following dynamical system:

$$\underline{\underline{M}}\, d^2\underline{u}\big/dt^2 + \underline{\underline{K}}\,\underline{u} = \underline{f}(t) \tag{1}$$

In (1), by using nodal shape functions $\mathbf{N}_i$, the displacement $\mathbf{u}(\mathbf{r},t)$ in V_c, is expressed as $\mathbf{u}(\mathbf{r},t)=\sum_{i=1:N_{dof}} u_i(t)\mathbf{N}_i(\mathbf{r})$; the number of degrees of freedom N_{dof} is equal to the product of the number of nodes and the three components of $\mathbf{u}(\mathbf{r},t)$. Then, $\underline{u}$ is the column vector made by the N_{dof} coefficients u_i. $\underline{\underline{M}}$ and $\underline{\underline{K}}$ are the sparse mass and stiffness matrix, respectively, while $f(t)$ is the vector of the nodal Lorentz forces defined as:

$$f_i(t) = \int_{V_c} \mathbf{N}_i \cdot \mathbf{J}(\mathbf{r},t)\times\mathbf{B}(\mathbf{r},t)\,d\tau, \quad i=1:N_{dof} \tag{2}$$

being **J** the current density and **B** the magnetic induction.

The electromagnetic system, in the magneto-quasi-stationary limit, with non magnetic materials, is described by the following weak formulation in terms of **J** in a Lagrangian coordinate system [6]:

$$\int_{V_c} \mathbf{W}\cdot\sigma^{-1}\mathbf{J}\,d\tau + \frac{d}{dt}\left[\int_{V_c}\mathbf{W}\cdot\left(\mathbf{A}[\mathbf{J}]+\mathbf{A}_s\right)d\tau\right]=0, \tag{3}$$

with $\mathbf{J}\in S, \forall \mathbf{W}\in S$, and

$$S=\left\{\mathbf{J}\in\mathbf{L}^2_{div}, \nabla\cdot\mathbf{J}=0 \text{ in } V_C,\ \mathbf{J}\cdot\hat{\mathbf{n}}=0 \text{ on } \partial V_C\right\} \tag{4}$$

Here $\mathbf{A}[\mathbf{J}]$ is the magnetic vector potential due to the eddy current density **J**, as given by the Biot-Savart law, σ is the electric conductivity and $\mathbf{A}_s$ is the vector potential due to the sources outside the conducting domain V_c.

By using solenoidal edge element shape functions $\mathbf{W}_k=\nabla\times\mathbf{T}_k$, **J** is expressed as

$$\mathbf{J}(\mathbf{r},t)=\sum_{k=1,N} I_k(t)\mathbf{W}_k(\mathbf{r}), \tag{5}$$

where N is the number of independent edges following from the application of the tree-cotree gauge [6]. Therefore, the integral equation is reduced to the following dynamical system [6]:

$$d\left(\underline{\underline{L}}\,\underline{I}\right)\big/dt + \underline{\underline{R}}\,\underline{I} + d\,\underline{E}^s\big/dt = \underline{0} \tag{6}$$

where

$$L_{ij}=\frac{\mu_0}{4\pi}\int_{V_c}\int_{V_c}\frac{\mathbf{W}_i(\mathbf{r})\cdot\mathbf{W}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d\tau d\tau',\quad R_{ij}=\int_{V_c}\mathbf{W}_i(\mathbf{r})\cdot\sigma^{-1}\mathbf{W}_j(\mathbf{r})d\tau$$

$$\frac{dE_i^s}{dt}=-\sum_{k=1,N_{dof}}F_{ik}^s\frac{du_k}{dt}-V_{0,i},\quad V_{0,i}=-\int_{V_c}\mathbf{W}_i(\mathbf{r})\cdot\frac{\partial\mathbf{A}_s(\mathbf{r},t)}{\partial t}d\tau$$

$$F_{ik}(t)=\int_{V_c}\mathbf{W}_i(\mathbf{r})\cdot\mathbf{N}_k(\mathbf{r})\times\mathbf{B}_s(\mathbf{r},t)d\tau \qquad (7)$$

Notice that, with $\mathbf{B}(\mathbf{r},t)\cong\mathbf{B}_s(\mathbf{r},t)$, being $\mathbf{B}_s(\mathbf{r},t)$ the field due to the sources, and $\mathbf{J}$ given by (5), $\underline{f}$ as defined by (2), is expressed as $\underline{f}(t)=-\underline{\underline{F}}^T(t)\underline{I}(t)$.

Finally, the coupled electro-magneto-mechanical dynamical system (1)-(6) can be written in a compact form as:

$$\underline{\underline{M}}\,d^2\underline{u}/dt^2+\underline{\underline{K}}\,\underline{u}+\underline{\underline{F}}^T(t)\underline{I}=\underline{0} \qquad (8)$$

$$d\left(\underline{\underline{L}}\,\underline{I}\right)/dt+\underline{\underline{R}}\,\underline{I}-\underline{\underline{F}}(t)\,d\underline{u}/dt=\underline{V}_0(t) \qquad (9)$$

The inductance matrix has been assumed to be unchanged with respect to time under the hypothesis of sufficiently small displacements.

This system is usually computationally very intensive, making them difficult to use when a large number of simulations are needed, like in the case of the design of the complex components of a Tokamak device. For this reason, we analyse the possibility of using a suitable modal expansion [7], [8]. In this approximation, we compute the N_{mode} dominant modes P^ks by solving the generalized eigenvalue problem $\left(-\omega_k^2\underline{\underline{M}}+\underline{\underline{K}}\right)\underline{P}^k=\underline{0}$. By using the classic linear transformation $\underline{u}=\left[\underline{P}^1\underline{P}^2...\underline{P}^{N\,mode}\right]\underline{x}=\underline{\underline{P}}\,\underline{x}$ and the orthogonality of $\underline{\underline{M}}$ and $\underline{\underline{K}}$ with $\underline{\underline{P}}$, we have:

$$\underline{\underline{m}}\,d^2\underline{x}/dt^2+\underline{\underline{k}}\,\underline{x}+\underline{\underline{P}}^T\underline{\underline{F}}^T(t)\underline{I}=\underline{0} \qquad (10)$$

$$\underline{\underline{L}}\,d\underline{I}/dt+\underline{\underline{R}}\,\underline{I}-\underline{\underline{F}}(t)\underline{\underline{P}}\,d\underline{x}/dt=\underline{V}_0(t) \qquad (11)$$

where $\underline{\underline{m}}$ and $\underline{\underline{k}}$ are diagonal matrices (i.e. $m_{ii}=\underline{P}^{iT}\underline{\underline{M}}\,\underline{P}^i$, $k_{ii}=\underline{P}^{iT}\underline{\underline{K}}\,\underline{P}^i$). If necessary, the system can be further simplified by introducing a similar expansion for solving (11).

These and other more sophisticated approaches will be outlined and compared in the full paper. Finally, the system (10), (11) is integrated in time by applying the Newmark's β method for solving (10) and the implicit method for (11) [9].

III. Results and Discussion

The proposed numerical approach has been validated by solving the TEAM-16 benchmark problem [1]. In this problem, a copper rectangular plate (L_x=115mm, L_y=40mm, L_z=0.3mm, electric conductivity σ=5.81×10^7 S/m, mass density ρ=8912kg/m^3, Young's modulus E=1.1×10^{11} Pa and Poisson's ratio ν=0.34), rigidly clamped at one hand (clamped length L_c=10 mm) is placed under a steady uniform magnet induction B_y and a pulsed magnetic field generated by a 27 turns circular coil. Its outer and inner diameters are 22 mm and 20 mm, respectively. The coil height is 24.2 mm. The distance between the plate and the coil is 9.5 mm and the coordinates of the coil center are (105mm, 0mm). The time dependent coil current is $i(t)$=800[exp(-500t)- exp(-600t)]A.

The mesh is made of 768 8-nodes brick elements (Fig. 1). The stiffness and mass matrices have been computed by the commercial code ANSYS. The time evolution of the deflection at the point A (105 mm, 7.5 mm, 0.15 mm) with B_y=0.3T is shown in Fig. 1. The results of the computation using 3 and 10 vibration modes are compared with the experiment, showing a good agreement.

In the full paper, the main features of this approach will be analyzed with a specific concern to fully 3D components of a fusion device. In particular, the importance of the magnetic damping will be assessed by analyzing the dynamic response of the vacuum vessel of a fusion device under the strong Lorentz forces due to the plasma current disruption. The complex geometry of the vacuum vessel represents a real challenging problem in this frame

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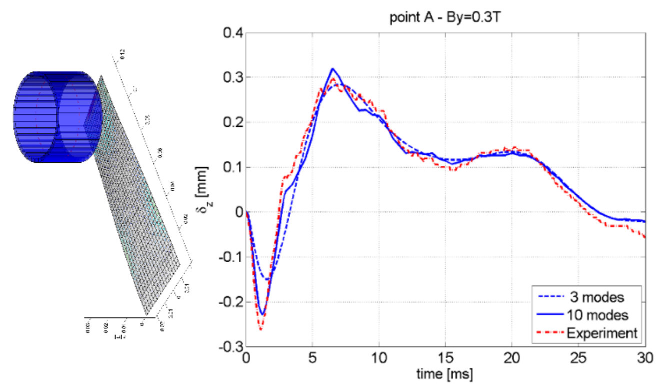

Fig.1.Left: Coil, deformed mesh of the plate and current density distribution at t=7ms. Right: Deflection at point A (108 mm 7.5 mm 0.mm) for B_y=0.3T.

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